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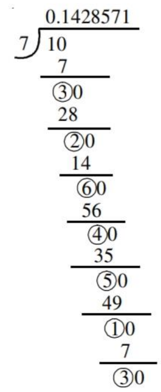
We are now ready to move on to the rational numbers whose decimal expansions are non-terminating and recurring. Once again, let us look at an example to see what is going on. We refer to Example 5, Chapter 1, from your Class IX textbook, namely, $\frac{1}{7}$. Here, remainders are 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, . . . and divisor is 7.



Notice that the denominator here, i.e., 7 is clearly not of the form 2^n5^m. Therefore, from Theorems 1.5 and 1.6, we know that $\frac{1}{7}$ will not have a terminating decimal expansion. Hence, 0 will not show up as a remainder (Why?), and the remainders will start repeating after a certain stage. So, we will have a block of digits, namely, 142857, repeating in the quotient of $\frac{1}{7}$.

What we have seen, in the case of $\frac{1}{7}$, is true for any rational number not covered by Theorems 1.5 and 1.6. For such numbers we have :

Theorem 1.7 : *Let $x = \frac{p}{q}$, where p and q are coprimes, be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).*

From the discussion above, we can conclude that *the decimal expansion of every rational number is either terminating or non-terminating repeating*.

EXERCISE 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

   (i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

   (v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

   (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.
3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can

3.8 Highest Common Factor

We can find the common factors of any two numbers. We now try to find the highest of these common factors.

What are the common factors of 12 and 16? They are 1, 2 and 4.

What is the highest of these common factors? It is 4.

What are the common factors of 20, 28 and 36? They are 1, 2 and 4 and again 4 is highest of these common factors.

Try These

Find the HCF of the following:

(i) 24 and 36 (ii) 15, 25 and 30

(iii) 8 and 12 (iv) 12, 16 and 28

The Highest Common Factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as Greatest Common Divisor (GCD).

The HCF of 20, 28 and 36 can also be found by prime factorisation of these numbers as follows:

2	20
2	10
5	5
	1

2	28
2	14
7	7
	1

2	36
2	18
3	9
3	3
	1

Thus, $20 = \boxed{2} \times \boxed{2} \times 5$

$28 = \boxed{2} \times \boxed{2} \times 7$

$36 = \boxed{2} \times \boxed{2} \times 3 \times 3$

The common factor of 20, 28 and 36 is 2(occuring twice). Thus, HCF of 20, 28 and 36 is $2 \times 2 = 4$.

EXERCISE 3.6

1. Find the HCF of the following numbers :
   (a) 18, 48 (b) 30, 42 (c) 18, 60 (d) 27, 63
   (e) 36, 84 (f) 34, 102 (g) 70, 105, 175
   (h) 91, 112, 49 (i) 18, 54, 81 (j) 12, 45, 75
2. What is the HCF of two consecutive
   (a) numbers? (b) even numbers? (c) odd numbers?